Faculty of Science, Technology, Engineering and Mathematics
M208 Pure mathematics

M208

TMA 03 2019J

Covers Book C **Cut-off date 16 January 2020**

You can submit your TMA either by post to your tutor or electronically as a PDF file by using the University's online TMA/EMA service.

Before starting work on the TMA, please read the document *Student guidance for preparing and submitting TMAs*, available from the 'Assessment' tab of the M208 website.

In the wording of the questions:

- *write down*, *list* or *state* means 'write down without justification' (unless otherwise stated)
- *find*, *determine*, *calculate*, *derive*, *evaluate* or *solve* means 'show all your working'
- *prove*, *show*, *deduce* or *verify* means 'justify each step'
- *sketch* means 'sketch without justification' and *describe* means 'describe without justification' (both unless otherwise stated).

In particular, if you use a definition, result or theorem to go from one line to the next, then you must state clearly which fact you are using – for example, you could quote the relevant unit and page, or give a Handbook reference. Remember that when you use a theorem, you must demonstrate that all the conditions of the theorem are satisfied.

The number of marks assigned to each part of a question is given in the right-hand margin, to give you a rough indication of the amount of time that you should spend on each part.

Your work should be written in a good mathematical style, as demonstrated by the exercise and worked exercise solutions in the study texts. You should explain your solutions carefully, using appropriate notation and terminology, defining any symbols that you introduce, and writing in proper sentences. Five marks (referred to as good mathematical communication, or GMC, marks) on this TMA are allocated for how well you do this.

Your score out of 5 for GMC will be recorded against Question 5. (You do not have to submit any work for this particular question.)

You should read the information on the front page of this booklet before you start working on the questions.

Question 1 (Unit C1) – 22 marks

(a) State which of the following matrices are in row-reduced form:

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 1 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix}.$$

For each matrix that is not in row-reduced form, explain why not and determine its row-reduced form. [5]

(b) (i) Solve the following system of linear equations by reducing its augmented matrix to row-reduced form.

$$\begin{aligned} x - y + z &= -1 \\ -x + 2y &= 1 \\ 2x + 4z &= -2 \end{aligned}$$

(ii) Verify your answer to part (b)(i) by substitution into the original equations. [6]

(c) (i) Find the determinant of the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 2 & 2 & 0 \end{pmatrix}.$$

(ii) Hence explain why $\mathbf{A}$ is invertible, and use row-reduction to find the inverse matrix, $\mathbf{A}^{-1}$, of $\mathbf{A}$.

(iii) Verify your answer to part (c)(ii) by matrix multiplication. [11]

Question 2 (Unit C2) – 25 marks

This question concerns the set $T = \{(x, y, z, 2x + y - 3z) : x, y, z \in \mathbb{R}\}$.

(a) Show that T is a subspace of $\mathbb{R}^4$. [5]

(b) Show that $S = \{(1, 0, 0, 2), (0, 1, 0, 1), (0, 0, 1, -3)\}$ is a basis for T, and state the dimension of T. [6]

(c) By applying the Gram–Schmidt orthogonalisation process to the basis S, or otherwise, find an orthogonal basis for T that includes the vector $(1, 0, 0, 2)$. [8]

(d) Show that the vector $\mathbf{u} = (1, 1, 1, 0)$ belongs to T, and find an expression for $\mathbf{u}$ in terms of the orthogonal basis for T that you found in part (c). [6]

Question 3 (Unit C3) – 26 marks

Let t be the function

$$\begin{aligned} t : \mathbb{R}^3 &\longrightarrow \mathbb{R}^3 \\ (x, y, z) &\longmapsto (x + 2z, 2x - y, y + 4z). \end{aligned}$$

(a) Use Strategy C14 from Unit C3 to show that t is a linear transformation. *[5]*

(b) Write down the matrix of t with respect to the standard basis for $\mathbb{R}^3$ for both the domain and the codomain. *[1]*

(c) Determine the matrix of t with respect to the basis $\{(1, 1, 0), (0, -1, 1), (1, 0, 2)\}$ for the domain and the standard basis for the codomain. *[2]*

(d) Find $\operatorname{Ker} t$, describe it geometrically, and state its dimension. *[4]*

(e) Find a basis for $\operatorname{Im} t$. State the dimension of $\operatorname{Im} t$, describe $\operatorname{Im} t$ geometrically, and give an equation for it. *[8]*

(f) Let s be the linear transformation

$$\begin{aligned} s : \mathbb{R}^3 &\longrightarrow P_3 \\ (a, b, c) &\longmapsto a - b + (b - c)x + (c - a)x^2. \end{aligned}$$

Find the matrix of s and the matrix of $s \circ t$ with respect to the standard basis for the domain $\mathbb{R}^3$ and the standard basis for the codomain P_3. *[6]*

Question 4 (Unit C4) – 22 marks

Use Strategy C25 from Unit C4 to find the standard form of the quadric with equation

$$2x^2 + 2y^2 + 4z^2 + 6xy + 2x - 2y = 1.$$

Which of the six types of quadric is this? *[22]*

Question 5 (Book C) – 5 marks

Five marks on this assignment are allocated for good mathematical communication in your answers to Questions 1 to 4.

You do not have to submit any extra work for Question 5, but you should check through your assignment carefully, making sure that you have explained your reasoning clearly, used notation correctly and written in proper sentences. *[5]*